Levelling Up Grades Through Skills

H2 Mathematics

Pure Mathematics

<u>Chapter 1:</u>
Functions

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Exam Requirements

- Be able to understand the concepts of function, domain and range.
- Calculate and find inverse functions and composite functions.
- Learn the conditions for the existence of inverse functions and composite functions.
- Apply Domain restriction to obtain an inverse function.
- Understand the relationship between a function and its inverse.

1. Relations and Functions

1.1 Relations

A relation from set A to set B is known as a _______ connecting elements in A (inputs) to elements in B (outputs). Set A is the _________ and set B is the ________ of the relation.

Domain represents the set of elements which are the ______.
Range represents the set of elements which are the _______.
Range is a ______ of the codomain.

1.2 Definition of Functions

Given a relation f: X —> Y, f is a function if each element in domain X maps to ________ element in the codomain Y.

The set X is the domain of f, denoted by D_f, and the set Y is the codomain of f.

If the element a $\in$ X is mapped to the element b $\in$ Y, then we write f(a)=b.
Two functions are _____ if they have the ____________________.

1.3 Representation of a function

$f : x \mapsto x^2 + x$, where $x \in \{1, 2, 3, 4\}$

$f(x) = x^2 + x$, where $x \in \{1, 2, 3, 4\}$

Domain of f =
Range of f =

1.4 Test if Relation exists as a Function

To test if a relation exists as a function graphically, a ________ line $x = k$, $k \in D_f$ drawn should cut graph f at only one point.

1.5 Types of Functions

A function is ____ if $f(x) = f(-x)$ for all values of x in its domain.

The graph of such a function is therefore ___________ about the ______.
The graph remains the same upon reflection in the y-axis.

A function is ___ if $f(-x) = -f(x)$ for all values of x in its domain.

The graph of such a function is therefore ___________ about the ______.
The graph remains the same upon a 180^o rotation about the origin.

A function f is _______ if no two elements in the given domain have the same output.

For $x_1, x_2 \in D_f$, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$
Equivalently, if $f(x_1) = f(x_2)$, then $x_1 = x_2$

Graphically, f is one-one if every horizontal line $y = k$, $k \in R_f$, cuts the graph of f at ________ point.

2. Inverse functions

2.1 Existence of an Inverse Function

Inverse function of f exists if f is _______.

2.2 Definition and Range of an Inverse Function

Let f be a one-one function. Then f has an inverse function f^{-1} defined by

Note that $f^{-1}(x) \neq \frac{1}{f(x)}$

2.3 Solving for Rule of Inverse Functions

i) Let y = f(x)
ii) Make x the subject of the formula in terms of y so that x = g(y) .
iii) From (i) and (ii), we have $x = f^{-1}(y) = g(y)$
Replacing y by x in $f^{-1}(y) = g(y)$, we obtain $f^{-1}(x) = g(x)$.

2.4 Graphical relationship between a Function and its Inverse

In general, if (a,b) is a point on the curve y=g(x) , then (b,a) is on the curve $y = g^{-1}(x)$ since $g(a) = b \Leftrightarrow g^{-1}(b) = a$.
Graphs of y = g(x) and $y = g^{-1}(x)$ are ___________ of each other in the line y = x.

3. Composite Functions

3.1 Definition of Composite Functions

Let f and g be two functions, the composite function gf is defined by
_______________ for all $x \in D_f$

$fg \neq gf$

The composite function ff is written as f^2
$f(f(x)) = ff(x) = f^2(x)$

3.2 Condition and Range of Composite Functions

For any given functions f and g, the composite function gf exists if
_____ of f ⊆ ______ of g,

For the composite function gf,
______ of gf = ______ of f,

When the domain of g = range of f, $D_g = R_f$
_____ of gf = _____ of g,

Inverse trigonometry function	**Notation**	**Equivalent form**	**Domain**	**Range of principal value**
arcsin	$y = \sin^{-1} x$	sin y =x	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
arccos	$y = \cos^{-1} x$	cos y =x	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
arctan	$y = \tan^{-1} x$	tan y =x	R	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Foundation Questions

Q1 Sketch the graphs of the following functions and state its range.

a) f : x |→ x^2 - 8x, x ϵ R, 1 ≤ x ≤ 10. Label the minimum point, intercept, f(1) and f(10). [3]

b) g : x |→ | ln(x) |, x ϵ R^+ [2]

c) h : x |→ 1 + e^x, x ϵ R, x < 2 [2]

d) k(x) = [2]

Q2 A curve C has equation $y^2 = 2x$, $x \in R_0^+$. By considering the graph of C, show that C does not represent a function. [2]

Q3 Sketch the graphs of the following functions. Determine its range and whether they are one-one.

a) $f : x \mapsto \sin x$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ [3]

b) $g : x \mapsto 2e^{-x^2}$, $x \in R$ [3]

Q4

a) Show that the inverse function of f, where $f(x) = 2x - 4$, $x \in R^+$, exists. Find $f^{-1}(x)$ and state the domain. [3]

b) Given that $g : x \mapsto 1 + e^{-x}$, $x \in R$, define its inverse function in similar form and state its range. [2]

Q5 The function f is defined as $f(x) = \sqrt{x+1}$, $x \geq -1$. Show that the inverse function of f exists. Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram, showing clearly the graphical relationship between the graphs. Hence, find the exact value of x which satisfies the equation f(x) = f-1(x) and deduce the solution set to $f(x) \leq f^{-1}(x)$. [4]

Q6 [4]

a) The functions f and g are defined by

$f(x) = \ln(x + 1), x > -1$

$g(x) = \sqrt{2x + 3}, x \geq -\frac{3}{2}$

Show that fg exists and define the function in a similar form. Show that gf does not exist

b) The function g is defined by $g(x) = \frac{4}{2-x}, x \in R, x \neq 0, 2$. Find an expression for $g^2(x)$ and $g^3(x)$. Hence, determine the value of $g(\frac{1}{2})$ and $g^{100}(\frac{1}{2})$. [4]

c) Let f and g be functions defined by $f(x) = e^{-x} + 1, x \in R^+$ and $g(x) = \ln(x), 1 < x < 2$. Determine whether the composite function fg exists. If it does, define the composite function in a similar form and find its range. [3]